

15. TIME AND WORK

IMPORTANT FACTS AND FORMULA

1. If A can do a piece of work in n days, then A's 1 day's work = $(1/n)$.
2. If A's 1 day's work = $(1/n)$, then A can finish the work in n days.
3. A is thrice as good a workman as B, then:
Ratio of work done by A and B = 3 : 1.
Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same Job. How long should it take both A and B, working together but independently, to do the same job? (IGNOU, 2003)

Sol. A's 1 hour's work = $1/8$

B's 1 hour's work = $1/10$

(A + B)'s 1 hour's work = $(1/8) + (1/10) = 9/40$

Both A and B will finish the work in $40/9$ days.

Ex. 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work? (Bank P.O. 2003)

Sol. (A + B)'s 1 day's work = $(1/4)$. A's 1 day's work = $(1/12)$.

B's 1 day's work = $((1/4) - (1/12)) = (1/6)$

Hence, B alone can complete the work in 6 days.

Ex. 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days

of 7 hours each. How long will they take to do it, working together 8 hours a day?

Sol. A can complete the work in $(7 \times 9) = 63$ hours.

B can complete the work in $(6 \times 7) = 42$ hours.

A's 1 hour's work = $(1/63)$ and B's 1 hour's work = $(1/42)$

$(A + B)$'s 1 hour's work = $(1/63) + (1/42) = (5/126)$

Both will finish the work in $(126/5)$ hrs.

Number of days. of $(42/5)$ hrs each = $(126 \times 5)/(5 \times 42) = 3$ days

Ex. 4. A and B can do a piece of work in 18 days; Band C can do it in 24 days A and C can do it in 36 days. In how many days will A, Band C finish it together and separately?

Sol. $(A + B)$'s 1 day's work = $(1/18)$ $(B + C)$'s 1 day's work = $(1/24)$

and $(A + C)$'s 1 day's work = $(1/36)$

Adding, we get: $2(A + B + C)$'s 1 day's work = $(1/18 + 1/24 + 1/36)$

$$= 9/72 = 1/8$$

$(A + B + C)$'s 1 day's work = $1/16$

Thus, A, Band C together can finish the work in 16 days.

Now, A's 1 day's work = $[(A + B + C)$'s 1 day's work] - $[(B + C)$'s 1 day work:

$$= (1/16 - 1/24) = 1/48$$

A alone can finish the work in 48 days.

Similarly, B's 1 day's work = $(1/16 - 1/36) = 5/144$

B alone can finish the work in $144/5 = 28 \frac{4}{5}$ days

And C's 1 day work = $(1/16 - 1/18) = 1/144$

Hence C alone can finish the work in 144 days.

Ex. 6. A is twice as good a workman as B and together they finish a piece in 18 days. In how many days will A alone finish the work?

Sol. $(A$'s 1 day's work): $(B$'s 1 days work) = $2 : 1$.

$(A + B)$'s 1 day's work = $1/18$

Divide $1/18$ in the ratio 2 : 1.

$$\therefore \text{A's 1 day's work} = (1/18 * 2/3) = 1/27$$

Hence, A alone can finish the work in 27 days.

Ex. 6. A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Sol. Ratio of times taken by A and B = 160 : 100 = 8 : 5.

Suppose B alone takes x days to do the job.

Then, 8 : 5 :: 12 : x = 8x = 5 x 12 = x = 7 1/2 days.

Ex. 7. A can do a piece of work in 80 days. He works at it for 10 days B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work?

Sol. Work done by A in 10 days = $(1/80 * 10) = 1/8$

Remaining work = $(1 - 1/8) = 7/8$

Now, 7/8 work is done by B in 42 days.

Whole work will be done by B in $(42 * 8/7) = 48$ days.

A's 1 day's work = $1/80$ and B's 1 day's work = $1/48$

(A+B)'s 1 day's work = $(1/80 + 1/48) = 8/240 = 1/30$

Hence, both will finish the work in 30 days.

Ex. 8. A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.

Sol : C's 1 day's work = $1/3 - (1/6 + 1/8) = 1/24$

A : B : C = Ratio of their 1 day's work = $1/6 : 1/8 : 1/24 = 4 : 3 : 1$.

A's share = Rs. $(600 * 4/8) = \text{Rs. } 300$, B's share = Rs. $(600 * 3/8) = \text{Rs. } 225$.

C's share = Rs. $[600 - (300 + 225)] = \text{Rs. } 75$.

Ex. 9. A and B working separately can do a piece of work in 9 and 12 days respectively, If they work for a day alternately, A beginning, in how many days, the work will be completed?

(A + B)'s 2 days' work = $(1/9 + 1/12) = 7/36$

Work done in 5 pairs of days = $(5 * 7/36) = 35/36$

Remaining work = $(1 - 35/36) = 1/36$

On 11th day, it is A's turn. $\frac{1}{9}$ work is done by him in 1 day.

$\frac{1}{36}$ work is done by him in $(9 \times \frac{1}{36}) = \frac{1}{4}$ day

Total time taken = $(10 + \frac{1}{4})$ days = $10 \frac{1}{4}$ days.

Ex 10 .45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

(45×16) men can complete the work in 1 day.

1 man's 1 day's work = $\frac{1}{720}$

45 men's 6 days' work = $(\frac{1}{16} \times 6) = \frac{3}{8}$

Remaining work = $(1 - \frac{3}{8}) = \frac{5}{8}$

75 men's 1 day's work = $\frac{75}{720} = \frac{5}{48}$

Now, $\frac{5}{8}$ work is done by them in 1 day.

$\frac{5}{8}$ work is done by them in $(\frac{48}{5} \times \frac{5}{8}) = 6$ days.

Ex:11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Soln: Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y .

Then, $2x + 3y = \frac{1}{10}$ and $3x + 2y = \frac{1}{8}$

Solving, we get: $x = \frac{7}{200}$ and $y = \frac{1}{100}$

$(2 \text{ men} + 1 \text{ boy})$'s 1 day's work = $(2 \times \frac{7}{200} + 1 \times \frac{1}{100}) = \frac{16}{200} = \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12 \frac{1}{2}$ days